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SOME COMPARISON THEOREMS IN DIFFERENCE EQUATIONS.

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ABSTRACT : In the present paper we discuss some comparison theorems approximate solutions, bounded and uniqueness in difference equations.

KEYWORDS : summation difference equation, second order, boundedness, approximate solutions.

INTRODUCTION : we consider successively some summation difference equations, summation operator, approximate solutions, bounded and uniqueness as well as maximal solutions in monotononic nondecreasing functions related to comparison theorems.

1. Comparison Theorems

Theorem 1.1.1 : Let $G \in C[I \times I \times R_+, R_+]$ and $G(t,s,u)$ be monotone nondecreasing in u for each (t, s) and $m(t) \le m_0(t) + \sum_{S=t_0}^{t-1} G(t,s,m(s)), t \ge t_0$

where $m \in C[I, R_+]$. Suppose $r(t)$ is the maximal solution of scalar summation equation

$$u_1(t) = u(t) + \sum_{s=t_0}^{t-1} g(t,s,u(s)) \tag{1.1.1}$$

Existing on I then the inequality $m(t_0) \le u_0(t_0)$ then

$$m(t) \le r(t) \tag{1.1.2}$$

Proof : Let $u(t,\in)$ be any solution of summation equation

$$u(t) = u_0(t) + \in + \sum_{s=t_0}^{t-1} G(t,s,u(s))$$

For $\in > 0$ sufficiently small. Since $\lim_{\in \to 0} u(t,\in) = r(t)$

$$m(t) < u(t,\in),\ t \ge t_0 \tag{1.1.3}$$

We observe that $m(t_0) < u(t_0 \in)$ and

$$u(t,\in) > u_0(t) + \sum_{t=t_0}^{t-1} G(t,s,u(s,\in))$$

We know, a) $K \in C[I \times I \times R_+, R_+], k(t,s,x)$ is monotone nondecreasing in x for each fixed (t,s) and one of the inequalities

$x(t) \le h(t) + \sum_{t=t_0}^{t-1} k(t,s,x(s))$ and $y(t) \ge h(t) + \sum_{s=t_0}^{t-1} k(t,s,y)$ is strict, where $x, y, h \in C[I, R]$

and $x(t_0) < y(t_0)$ then $x(t) < y(t)$, $t \ge t_0$.

By applying this fact we obtain $m(t) < u(t, \in)$, $t \ge t_0$

Theorem 1.1.2 Let $k \in C[I \times I \times R_+, R_+]$, $k(t,s,x)$ be monotonic nondecreasing in x for each (t, s) and

$$x(t) \le x_0(t) + \sum_{s=t_0}^{t-1} k(t,s,x(s)) \qquad 1.1.4$$

Where $x, x_0 \in C[I, R_+]$. Assume that $r(t)$ is the maximal solution of

$$u(t) = x_0(t) + \sum_{s=t_0}^{t-1} k(t,s,x(s)) \qquad 1.1.5$$

Existing on $[t_0, \infty)$ then

$$x(t) \le r(t), \quad t \ge t_0. \qquad 1.1.6$$

Proof : Define $F(t,s,y) = k(t,s,\sup[y, x(t)])$ 1.1.7

For any two element $x, y \in R_+, x \le y$ if and only if $x_i \le y_i$ for $i = 1,2,3,4\ldots$n.

If $A \subset R^n$ then there exists a sup A with respective relation $x \le y,\ x_i \le y_i$

$$\sup\{x, y] = Z = (z_1, z_2 \ldots\ldots, z_n) \qquad 1.1.8$$

Where $Z_i = \max(x_i, y_i)$ where x_i, y_i is component of x,y $x(t) \le \sup[y, x(t)]$ and it follows from the monotonicity of K and (1.1.7)

$F(t, x, y) \ge K(t, s, x(t))$ for each y ……………. 1.1.9

Let $r^*(t)$ be the maximal solution of

$$u(t) = x_0(t) + \sum_{s=t_0}^{t-1} F(t,s,u(s))$$

existing on $[t_0, \infty)$ by applying (1.1.7) and (1.1.4) we obtain

$$r^*(t) = x_0(t) + \sum_{s=t_0}^{t-1} F(t,s,r^*(s)) \ge x_0(t) + \sum_{t=t_0}^{t-1} k(t,s,x(s)) \ge x(t) \qquad 1.1.10$$

From $\ge x(t)$ 1.1.10 and (4.1.8) it is clear that. (4.1.10)

$\sup[t^*(t), x(t)] = r^*(t)$

By (1.1.7)

$F(t, x, r^*(t)) = K(t, s, r^*(t))$

Therefore $r^*(t)$ is also the maximal solution of (1.1.5)

$\therefore x(t) \le r(t), t \ge t_0$

2. APPROXIMATE SOLUTIONS, BOUNDED AND UNIQUENESS

Definition 2.1.1. Let $x \in C[I, R_+]$ and satisfy

$$\| x(t) - x_0(t) - \sum_{t=t_0}^{t-1} K(t, s, x(s)) \| \le \delta(t) \qquad (2.1.1)$$

Where $\delta \in C[I, R_+]$ then $x(t)$ is said to be $\delta -$ approximate

Solution of summation equation $x(t) = x_0(t) + \sum_{s=t_0}^{t-1} k(t, s, x(s))$

Theorem 5.1.1 Assume that

a) $k \in C[I \times I \times R_+, R_+], G \in C[I \times I \times R_+, R_+]$

$G(t, s, u)$ is monotonic non decreasing in u for each (t, s) and

$$\| k(t, s, x) - k(t, s, y) \| \le G(t, s,) \| x - y \| \qquad (2.1.2)$$

b) $x(t, \delta)$ is a $\delta -$approximate solution of

$x(t) = x_0(t) + \sum_{s=t_0}^{t-1} k(t, s, x(s))$ where $\delta \in C[I, R_+]$

c) $r(t)$ is the maximal solution of (2.1.3)

$$u(t) = \delta(t) + \sum_{s=t_0}^{t-1} G(t, s, u(s))$$

erxisting on $[t_0, \infty)$

Then , if y(t) is an solution of (2.1.3) existing on $[t_0, \infty)$

the $\| x(t, \delta) - y(t) \| \le r(t), t \ge t_0$

Proof : Consider the function

$$m(t) = \| x(t, \delta) - y(t) \|$$

Where $x(t, \delta)$ and $y(t)$ are approximate solution and solution of (2.3.1) respectively then using (2.1.1), (2.1.2) we obtain

$$m(t) = \| x(t, \delta) - x_0(t) - \sum_{s=t_0}^{t-1} k(t, s, x(s, \delta))$$

$$+\sum_{s=t_0}^{t-1} k(t,s,x(s,\delta)) - k(t,s,y(s)) \| \le \delta(t) + \sum_{s=t_0}^{t-1} G(t,s,m(s))$$

Theorem 2.1.2. Assume that $K \in C[I \times I \times R_+, R_+], G \in C[I \times I \times R_+, R_+], G(t,s,u)$ is monotonic non-decreasing in u for each (t,s) and $\| K(t,s,x) \| < G(t,s,\| x \|)$ (2.1.4)

a) r(t) is the maximal solution of (4.4.1) existing on $[t_0, \infty)$

b) $x(t)$ is any solution of (2.3.1) existing on such that $\| x_0(t) \| \le u_0(t)$

then $\| x(t) \| \le r(t), \ t \ge t_0$ (2.1.5)

Proof : Let $m(t) = \| x(t) \|$ then the summation inequality

$$m(t) \le \| x_0(t) \| + \sum \| K(t,s,x) \| \le u_0(t) + \sum_{s=t_0}^{t-1} G(t,s,m(s))$$

By theorem 1.1.1 we obtain

$$\| x(t) \| \le r(t), \ t \ge t_0$$

Theorem 2.1.3. Suppose that

a) $k_1, k_2 \in C[I \times I \times R_+, R_+], G \in C[I \times I \times R_+, R_+], G(t,s,u)$

is monotonic nondecreasing in u for each (t,s) and

$\| K_1(t,s,x) - k_1(t,s,y) \| \le G(t,s,)(x-y) \|$ (2.1.6)

b) $x_0, y_0 \in C[I, R_+]$ and $x(t), y(t)$ are any two solution of

$$x(t) = x_0 + \sum_{s=t_0}^{t-1} k_1(t,s,x(s))$$

$$y(t) = y_0 + \sum_{s=t_0}^{t-1} k_2(t,s,y(s)) \text{ respectively}$$

c) r(t) is the maximal solution of (5.4.1) such that

$\| x_0(t) - y_0(t) \| \le r(t), t \ge t_0$

Proof : Firstly setting $m(t) = \| x(t) \|$ and using (5.1.6)

We obtain

$$m(t) \le \| x_0(t) - y_0(t) \| + \sum_{s=t_0}^{t-1} \| k_1(t,s,x(s)) - k_2(t,s,y(s)) \|$$

$$\le u_0(t) + \sum_{s=t_0}^{t-1} G(t,s,m(t))$$

From theorem 4.1 we get

$\| x(t)-y(t) \| < r(t), t \geq t_0$

Theorem : 2.1.4 Assume that $K_1, K_2 \in C[I \times I \times R_+, R_+]$ and $G \in C[I \times I \times R_+, R_+]$, $G(t,s,u)$ is monotonic non decreasing in u for each (t,s) and $\| K_1(t,s,x) - K_2(t,s,y) \| \leq G(t,s, \| x-y \|)$

a) $x_0, y_0 \in C[I, R_+]$ and $x(t), y(t)$ are two solution of

$$x(t) = x_0(t) + \sum_{t=t_0}^{t-1} K_1(t,s,x(s))$$

$$y(t) = y_0(t) + \sum_{t=t_0}^{t-1} K_2(t,s,x(s)) \text{ respectively}$$

b) $r(t)$ is the maximal solution of $u(t) = u_0(t) + \sum_{t=t_0}^{t-1} G(t,s,u(s))$ such that

$\| x_0(t) - y_0(t) \| \leq u_0(t), \quad t \geq t_0$

under these assumptions we obtain

$\| x(t) - y(t) \| \leq r(t), \; t \geq t_0$

proof : Let $m(t) \leq \| x_0(t) - y_0(t) \| + \sum_{s=t_0}^{t-1} \| K_1(t,s,x(s)) - K_2(t,s,y(s)) \|$

$$\leq u0(t) + + \sum_{s=t_0}^{t-1} G(t,s,m(s))$$

Theorem 2.1.5 Suppose that $G \in C\{[t_0, t_0+a] \times [t_0, t_0+a] \times R_+, R_+\}, G(t,s,o) = 0$

a) $G(t,s,u)$ is monotone non decreasing in u for each (t,s,) and $u(t)=0$ the only solution of summation equation

$$u(t) = \sum_{t=t_0}^{t-1} G(t,s,u(s)) \text{ on } [t_0, t_0+a] \qquad (5.1.4.1)$$

b) $K \in C\{[t_0, t_0+a] \times [t_0, t_0+a] \times R_+, R_+\}$ and

$\| K(t,s,x) - K(t,s,y) \| \leq G(t,s), \| x-y \|)$

then there exists at most one solution of (2.1.3) on to

Proof : Let $x(t)$, $y(t)$ be two solution of (2.1.3) existing on $[t_0, t_0+a]$

Let $m(t) = \| x(t) - y(t) \|$ then

$$m(t) \leq \sum_{t=t_0}^{t-1} G(t,s,m(s))$$

By applying theorem 4.1 we obtain

$$m(t) \leq r(t), t \geq t_0 , \ t \geq t_0$$

Where $r(t)$ is the maximal solution (2.1.4.1) since $m(t_0) = 0$ and is only solution of (2.1.4.1) therefore there exists at most one solution of (2.1.3) on to $t_0 \leq t \leq t_0 + a$

REFERENCE

1. Ahmad, S, Rama Mohana Rao, M: Theory of Ordinary Differential Equations. With Applications in Biology and Engineering. Affiliated East-West Press Pvt. Ltd., New Delhi (1999)
2. Baxley, JV: Global existence and uniqueness for second-order ordinary differential equations. J. Differ. Equ. 23(3), 315-334 (1977); Stanford University Press, Stanford, Calif. (1963).
3. Burton, TA: Stability and Periodic Solutions of Ordinary and Functional Differential Equations. Corrected version of the 1985 original. Dover, Mineola (2005)
4. Constantin, A: Global existence of solutions for perturbed differential equations. Ann. Mat. Pura Appl. (4) 168, 237-299 (1995)
5. Driver, RD: Existence and stability of solutions of a delay-differential system. Arch. Ration. Mech. Anal. 10, 401-426 (1962)
6. Fujimoto, K, Yamaoka, N: Global existence and nonexistence of solutions for second-order nonlinear differential equations. J. Math. Anal. Appl. 411(2), 707-718 (2014)
7. Grace, S.R., Lalli, BS: Asymptotic behavior of certain second order integro-differential equations. J. Math. Anal. Appl. 76(1), 84-90 (1980)
8. Graef, J, Tunç, C: Continuability and boundedness of multi-delay functional integro-differential equations of the second order. Rev. R. Acad. Cienc. Exactas Fís. Nat, Ser. A Mat. 109(1), 169-173 (2015)
9. Kalmanovskii, V, Myshkis, A: Introduction to the Theory and Applications of Functional-Differential Equations. Mathematics and Its Applications, vol. 463. Kluwer Academic, Dordrecht (1999)
10. Krasovskii, NN: Stability of Motion. Applications of Lyapunov's Second Method to Differential Systems and Equations with Delay. Stanford University Press, Stanford (1963)
11. Miller, RK: Asymptotic stability properties of linear Volterra integro-differential equations. I. Differ. Equ. 10, 485-506 (1971)

12. Miller, RK, Michel, AN: Ordinary Differential Equations. Academic Press, New York (1982)
13. Mustafa, GO, Rogovchenko, YV: Global existence of solutions with prescribed asymptotic behavior for second-order nonlinear differential equations. Nonlinear Anal., Theory Methods Appl. 51(2), 339-368 (2002)
14. Mustafa, GO, Rogovchenko,YVglobal existence of solution for a class of nonlinear differential equations. Appl. Math.Lett 16(5), 753-758 (2003).
15. Napoles Valdes, JE: A note on the boundedness of an integro-differential equation. Quaest. Math. 24(2), 213-216 (2001)
16. Ogundare, BS, Ngcibi, S, Murali, V: Boundedness and stability properties of solutions to certain second-order differential equation. Adv. Differ. Equ. Control Process. 5(2), 79-92 (2010)
17. Reissig, R, Sansone, G, Conti, R: Non-linear Differential Equations of Higher Order. Translated from the German. Noordhoff International Publishing, Leyden (1974)
18. Tidke, HL:: Global existence of solutions for nonlinear integral equations of second order. J. Appl. Funct, Anal. 5(1), 113-120 (2010)
19. Tiryaki, A, Zafer, A: Global existence and boundedness for a class of second-order nonlinear differential equations. Appl. Math. Lett. 26(9), 939-944 (2013)
20. Tunç, C Boundedness results for solutions of certain nonlinear differential equations of second order. J. Indones. Math. Soc. 16(2), 115-126 (2010)
21. Tunç, C: Some new stability and boundedness results of solutions of Liénard type equations with a deviating argument. Nonlinear Anal. Hybrid Syst. 4(1), 85-91 (2010)
22. Tunç, C: Stability and boundedness of solutions of non-autonomous differential equations of second order. J. Comput. Anal. Appl. 13(6), 1067-1074 (2011)
23. Tunç, C: Uniformly stability and boundedness of solutions of second order nonlinear delay differential equations Appl, Comput. Math. 10(3), 449-462 (2011)
24. Tunç, C: On the stability and boundedness of solutions of a class of nonautonomous differential equations of second order with multiple deviating arguments. Afr. Math. 23(2), 249-259 (2012)
25. Tunç, C: A note on the stability and boundedness of non-autonomcus differential equations of second order with a variable deviating argument. Afr. Math. 25(2), 417-425 (2014)

26. Tunç, C, Tunç, O: A note on certain qualitative properties of a second order linear differential system. Appl. Math. Inf. Sci. 9(2), 953-956 (2015)
27. Yoshizawa, T: Stability Theory by Liapunov's Second Method. Publications of the Mathematical Society of Japan, vol. 9. The Mathematical Society of Japan, Tokyo (1966)
28. Wu, C, Hao, S, Xu, C: Global existence and boundedness of solutions to a second-order nonlinear differential system. J. Appl. Math. 2012, Article ID 603783 (2012)
29. Yin, Z Global existence and boundedness of solutions to a second order nonlinear differential system. Studia Sci. Math. Hung. 41(4), 365-378 (2004)